Site index curves and growth model for Mediterranean maritime pine (*Pinus pinaster* Ait.) in Spain

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**Abstract**

A stand growth model for Mediterranean maritime pine in the Iberian Peninsula (*Pinus pinaster* Ait.) is presented. The model consist of a site index submodel developed from 281 stem analysis and validated with the data from 92 permanent sample plots. Three height growth equations in difference form are tested and the Bailey and Clutter (1974) function is considered appropriate due to its good performance with both fitting and validation data. A compatible growth and yield submodel is then elaborated with the data from the Forest Research Centre (CIFOR-INIA) permanent sample plots network. The future state of the stand is determined by the current state, characterized by basal area, age and density. The model is completed with a control function that predicts the diameter after thinning and a mortality rate for non-thinned intervals. The global model was validated with data of 13 independent permanent thinning plots from two experimental sites. The simulation of the long-term projection of volume, basal area and DBH after thinning shows certain overestimation in diameter after thinning and in volume. However, the results show errors lower than 5% and little bias.

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**Keywords:** Site index; Growth model; Difference equations; Simultaneous system; *Pinus pinaster* Ait

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1. Introduction

The most extensive conifer forests in Spain are *Pinus pinaster* Ait., covering approximately 1,200,000 ha, including natural and artificial stands. The great ecological variability of this species and the geographical isolation among its populations imply the existence of several geographic races, which present different genetic characteristics and behaviour. Maritime pine races can be divided into three groups (Barandat and Murpeau, 1988): the Atlantic group, which coincides with the so-called ssp. *atlantica* H. del Villar (Milov, 1967; Tutin and Heywood, 1964); the European Mediterranean group; and the Maghreb group; the last two groups constitute the Mediterranean subspecies, ssp. *pinaster* (*P. mesoaeensis* F. et Gaussen).

The Atlantic group of maritime pine shows a higher growth rate than the other groups. Although the area...
occupied by the Mediterranean subspecies in Spain represents more than two-thirds of the total distribution, the total annual growth in the Atlantic group is 1.5 times higher, resulting in similar standing volumes at national level in both subspecies (51 and 58 million m³ for the Atlantic and Mediterranean subspecies, respectively (DGCN, 1998). The high productivity of the *atlantica* subspecies gives its forests an important economical value. Therefore, the majority of the recent reports on growth and yield models of the species have been focused on the Atlantic group (Falcão, 1997; Álvarez-González et al., 1999, 2002; Rodríguez-Soalleiro et al., 2000; Timbal, 2002).

In spite of the low productivity of the Mediterranean subspecies, it plays a very important role in Spain’s forest ecosystems, with multi-purpose use like wildlife, hunting, recreation, protection, etc. Especially important are the stands reforested in the last 60 years in order to prevent soil erosion, which represent more than half of the species’ area (Alía et al., 1996).

In the last few years, some partial aspects of the growth and yield of Mediterranean maritime pine have been studied, for example, site productivity (Gonzalo and Sánchez, 1997), biomass production (Montero et al., 1999), resin yield (Nanos et al., 2000, 2001) and the effect of the stand structure on growth (Bravo and Guerra, 2003). However, the only growth models available for maritime pine in the Spanish Mediterranean area are the yield tables for maritime pine stands in the Central Mountain Range (García and Gómez, 1989). Moreover, most of the aforementioned studies concern small geographic areas of the subspecies. Thus, it is necessary to develop a flexible growth model which could facilitate the decision making, in order to guarantee the sustainable management.

The modelling level is often dictated by available data, desired level for prediction and time horizon for projection (Burkart, 2003). Data collected in the net of permanent sample plots established in the sixties for studying growth in *P. pinaster* stands in Spain are limited for the development of an individual-tree model because of the scarce information measured at tree-level. On the other hand, the low economical value and the extensive character of the management of maritime pine stands in the Mediterranean area make it advisable to choose a stand modelling level which requires only basic input data, such as site index, density, basal area, etc.

Yield tables are static stand models, frequently based on fully-stocked stands, which do not accurately reflect the stand evolution under different thinning intensities and with multiple initial data. On the other hand, growth models which include a good description of the state of the stand at any point in time, as well as the change rates as a function of this state are able to adequately predict the stand development under different assumptions (García, 1994). In stand growth and yield models, the state is usually described by top height, number of trees and basal area; volume is also considered sometimes (Borders and Bailey, 1986; García, 1994; Fang et al., 2001; García and Ruiz, 2002).

Taking into account these premises, the aim of this work is to develop a stand growth model for Mediterranean maritime pine in Spain. First, site index curves will be fitted in order to determine site index of the stands and to predict the top height growth. After that, a stand growth and yield model will be developed, including a thinning function to simulate different silvicultures and a mortality function to estimate natural mortality when thinning is not performed.

2. Material and methods

2.1. Data

Data were obtained from 92 permanent sample plots (PSP) of the Forest Research Centre (CIFOR-INIA). Plots from natural and sowed forests were established according to Forestry Commission recommendations (Hummel et al., 1959), covering different ages and quality sites.

This network was established in the early 1960’s to assess forest productivity by re-measuring at 5 year intervals the diameter of all trees at breast height and a sub-sample of heights in order to obtain the mean and top height defined as the average height of the 100 thickest stems per hectare (Assmann, 1971). Height-diameter functions were adjusted per plot and per inventory to estimate non-measured heights. Tree volume was calculated from tree diameter and height using a tree volume equation (Martínez-Millán et al., 1993). No silvicultural operations were made in plots except in the second inventory, when a low thinning was carried out.
Next to each plot and in similar stand conditions, two or three dominant trees were felled at installation time for stem analysis purposes, resulting in 281 trees. In each tree, a measurement was recorded 30 cm above ground and every 2.5 m.

To evaluate the results, data of 13 independent experimental thinning permanent plots were used. Seven of these plots are located in Vado Forest and six in Atienza Forest, both of them in the Central Mountain Range. Table 1 shows the descriptive statistics of fitting and validation data set.

2.2. Methods

2.2.1. Site index curves

2.2.1.1. Fitting. Site index curves usually needs a base-age to estimate the site index value. Base-age selection may be problematic as results in errors and low flexibility of the shape of the curves and may, also, influence the accuracy of site index estimates (Heger, 1973). To avoid this problem base-age invariant equations were preferred.

Several methods have been proposed in the literature to generate site index curves with desirable characteristics: asymptote, inflection point, base-age invariance or logical behavior among others (Goetz and Burk, 1992; Elfving and Kiviste, 1997). Bailey and Clutter (1974) used difference equation of the form \( H_2 = f(T_2, T_1, H_1) \) where \( H_2 \) and \( H_1 \) are heights at age \( T_2 \) and \( T_1 \), respectively. This method gathers many of the desirable characteristics of a well-behaved site index curve. In this study, the Bailey and Clutter (1974) function, the Richards (1959) function and the McDill and Amateis (1992) function were considered as candidate functions to be differentiated. There are as many difference equations as parameters in the original function, which may come from height-age equation or differential forms (Cao, 1993). In this work, only those parameters related to growth rate and asymptote were considered as free. In accordance with this, we have fitted five difference form models to our data. Table 2 shows the difference form of all functions tested.

According to Huang (1997), there are six types of data structures that can be used to fit difference equation growth models: The longest non-descending growth interval (I), the longest growth intervals (II), the non-overlapping, non-descending growth interval (III); the non-overlapping growth intervals (IV); the all possible non-descending growth intervals (V) and the all possible growth intervals (VI); and found that structure type VI provided the most stable and consistent results, as Goetz and Burk (1992) noted before. In this study, 281 stem analysis and all possible growth intervals were used resulting in 9633 pairs of observations.

Fitting coefficients from the data of temporal series, such as stem analysis may present autocorrelation. This means that the error term is not

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Table 1
Initial mean values of forest attributes in PSP and stem analysis

<table>
<thead>
<tr>
<th>Data</th>
<th>Variable</th>
<th>Units</th>
<th>No. plots/stems</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stem Analysis</td>
<td>Age</td>
<td>year</td>
<td>281</td>
<td>44.64</td>
<td>22.40</td>
<td>15</td>
<td>125</td>
</tr>
<tr>
<td></td>
<td>Total height</td>
<td>m</td>
<td>281</td>
<td>11.08</td>
<td>3.32</td>
<td>4.3</td>
<td>26</td>
</tr>
<tr>
<td>PSP</td>
<td>Age</td>
<td>year</td>
<td>92</td>
<td>44.20</td>
<td>20.94</td>
<td>18</td>
<td>127</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>stems ha(^{-1})</td>
<td>92</td>
<td>1001.63</td>
<td>730.54</td>
<td>189</td>
<td>4960</td>
</tr>
<tr>
<td></td>
<td>Volume</td>
<td>m(^3) ha(^{-1})</td>
<td>92</td>
<td>187.38</td>
<td>114.64</td>
<td>20.11</td>
<td>491.78</td>
</tr>
<tr>
<td></td>
<td>Dgθt</td>
<td>cm</td>
<td>92</td>
<td>22.57</td>
<td>7.05</td>
<td>9.16</td>
<td>45.28</td>
</tr>
<tr>
<td></td>
<td>Basal area</td>
<td>m(^2) ha(^{-1})</td>
<td>92</td>
<td>32.97</td>
<td>13.70</td>
<td>7.29</td>
<td>64.73</td>
</tr>
<tr>
<td></td>
<td>Top height</td>
<td>m</td>
<td>92</td>
<td>12.10</td>
<td>3.58</td>
<td>5.36</td>
<td>25.56</td>
</tr>
<tr>
<td>Validation data set</td>
<td>Age</td>
<td>year</td>
<td>13</td>
<td>29.61</td>
<td>5.04</td>
<td>23</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>Stem stock</td>
<td>stems ha(^{-1})</td>
<td>13</td>
<td>1277.7</td>
<td>171.71</td>
<td>910</td>
<td>1500</td>
</tr>
<tr>
<td></td>
<td>Volume</td>
<td>m(^3) ha(^{-1})</td>
<td>13</td>
<td>154.79</td>
<td>50.11</td>
<td>88.01</td>
<td>233.56</td>
</tr>
<tr>
<td></td>
<td>Dgθt</td>
<td>cm</td>
<td>13</td>
<td>18.57</td>
<td>2.19</td>
<td>15.17</td>
<td>21.70</td>
</tr>
<tr>
<td></td>
<td>Dgat</td>
<td>cm</td>
<td>13</td>
<td>19.11</td>
<td>1.97</td>
<td>15.79</td>
<td>21.79</td>
</tr>
<tr>
<td></td>
<td>Basal area</td>
<td>m(^2) ha(^{-1})</td>
<td>13</td>
<td>34.22</td>
<td>5.16</td>
<td>26.21</td>
<td>43.88</td>
</tr>
<tr>
<td></td>
<td>Top height</td>
<td>m</td>
<td>13</td>
<td>9.99</td>
<td>2.32</td>
<td>6.73</td>
<td>14.06</td>
</tr>
</tbody>
</table>

dgθt: quadratic mean square diameter at breast height before thinning; dgat: quadratic mean diameter after thinning.
Table 2
Candidate functions to site index model

<table>
<thead>
<tr>
<th>Original function</th>
<th>Free parameter</th>
<th>Difference form</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Richards (1959) $Y = A(1 - \exp(-kT))^\gamma$</td>
<td>$A$</td>
<td>$H_2 = H \left(1 - \frac{\exp(k \times T_2)}{\exp(k \times T_1)}\right)^\gamma$ (1.1)</td>
</tr>
<tr>
<td></td>
<td>$k$</td>
<td>$H_2 = A \left(1 - \left(\frac{H_1}{A}\right)^\frac{1}{\gamma} \right)^\frac{T_2}{T_1}$ (1.2)</td>
</tr>
<tr>
<td>(2) Bailey and Clutter (1974)* $\ln Y = a + b \times T^c$</td>
<td>$a$</td>
<td>$H_2 = H_1 \exp\left(b \times \left[T_2^c - T_1^c\right]\right)$ (2.1)</td>
</tr>
<tr>
<td></td>
<td>$b$</td>
<td>$H_2 = \exp\left(a + (\ln(H_1) - a) \left(\frac{T_2}{T_1}\right)^c\right)$ (2.2)</td>
</tr>
<tr>
<td>(3) McDill and Amateis (1992)</td>
<td>No integral form</td>
<td>$H_2 = \frac{M}{1 - \left(1 - \frac{M}{H_1}\right) \left(\frac{T_1}{T_2}\right)^\alpha}$ (3)</td>
</tr>
</tbody>
</table>

* Cao’s formulation of the Bailey and Clutter model (in Cao, 1993). $H_1$, $H_2$ are dominant heights at time $T_1$ and $T_2$, respectively; $A$, $a$, $b$, $c$, $k$, $m$, $M$ and $\alpha$ are parameters to be obtained. Parameters $A$ and $a$ are the asymptote, $k$, $b$ and $c$ are related to growth rate and $m$ is a specific growth rate/shape parameter (Amaro et al., 1998).

independent and does not follow a normal distribution. To avoid this, fitting was performed in a two step procedure using PROC NLIN of SAS/STAT module (SAS, 1990). In the first step the parameters are fitted according to ordinary non-linear least squares. After that, the error term is expanded and the model is fitted again. The expansion of the model leads to a first order auto-regressive procedure in order to make the error term independent. Let the difference model be:

$$H_{ij} = f(T_i, H_{ij}, T_j, \beta) + e_{ij}$$

where $H_{ij}$ is a prediction of height $i$ by using height $j$, $T_i$ is age $i$, $T_j$ is age $j$; $\beta$ is the vector of parameters and $e_{ij}$ is the error term that is not independent and not identically distributed (Monserud, 1984; Goelz and Burk, 1992,1996). This error is expanded in the following way:

$$e_{ij} = \rho \times e_{i-1,j} + \gamma \times e_{i,j-1} + \epsilon_{ij}$$

where $\rho$ is the auto-correlation between the residual of estimation $H_{ij}$ and the residual that corresponds to estimation $H_{i-1,j}$ when $H_j$ is used as a predictor variable, $\gamma$ is the relationship between the current residual and the residual from estimating $H_i$ using $H_{j-1}$ as a predictor variable. $\epsilon_{ij}$ represents independent errors, with mean value equalling zero and constant variance equalling $\eta^2$. $\rho$ and $\gamma$ have no further use in the site index model as $e_{i-1,j}$ and $e_{i,j-1}$ values is not observable without stem analysis (Monserud, 1984).

2.2.1.2. Validation. For site index model validation purposes, re-measured permanent sample plots were used. The significance of the bias was studied Absolute mean residual, AMRES (Eq. (2)), root of mean square error (RMSE) (Eq. (3)) and model efficiency, EF (Eq. (4)) are measures of the accuracy of the model (Soares et al., 1995; Gadow et al., 2001) and they were computed with the fitting data set as well as with the independent data set for validation purposes. The percentage of AMRES and RMSE was also computed. The model that accounted for the best accuracy in fitting and validation data was chosen.

$$AMRES = \frac{\sum |y_i - \hat{y}_i|}{n}$$ (2)

$$RMSE = \sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n - 1 - p}}$$ (3)

$$EF = 1 - \frac{\sum (y_i - \hat{y})^2}{\sum (y_i - \bar{y})^2}$$ (4)
where $y_i$ is observed value, $\hat{y}$ is estimated value, $\bar{y}$ is average observed value, $n$ is the number of data and $p$ is the number of parameters in the model.

2.2.2. Growth and Yield model, thinning control function and mortality rate

2.2.2.1. Fitting. A logical treatment of growth and yield should take into account a compatible model in which yield is obtained by the summation of the predicted growth (Clutter, 1963), so growth and yield are not considered as independent phenomena in successive growth intervals. The use of growth and yield equations may result in errors such as non-independence of parameters between equations in the model; on the other hand, permanent plots with successive measurements on the same plot do not constitute statistically independent observations (Sullivan and Clutter, 1972). To avoid this, the aforementioned authors proposed a compatible system of equations relating projected stand basal area and volume to initial stand age, projected age, site index and initial basal area (Eqs. (5)–(7)).

\[
\ln V_i = b_0 + b_1 \cdot S + \left( \frac{b_2}{t_i} \right) + b_3 \cdot \ln G_i
\]

(5)

\[
\ln G_{i+5} = \left( \frac{t_i}{t_{i+5}} \right) \times \ln G_i + \left[ (a_0 + a_1 \times S) \times \left( 1 - \frac{t_i}{t_{i+5}} \right) \right]
\]

(6)

\[
\ln V_{i+5} = \left[ b_0 + b_1 \times S + \left( \frac{b_2}{t_{i+5}} \right) \right.

+ b_3 \left( \frac{t_i}{t_{i+5}} \right) \times \ln (G_i) + b_3 \times (a_0 + a_1 \times S)

\times \left( 1 - \frac{t_i}{t_{i+5}} \right) \left. \right]
\]

(7)

where $t_i$ is stand age at ith measurement, $V_i$ is stand volume at age $i$, $G_i$ is basal area at age $i$, $S$ is site index, (in our case, dominant height at age 80 obtained by previously fitted site index curves), $a_0$, $a_1$, $b_0$, $b_1$, $b_2$, and $b_3$ are parameters to be obtained.

To fit the compatible system, a PROC MODEL procedure of SAS/ETS (SAS, 1999) system was performed. Three stage least squares (3SLS) method was used in order to obtain consistent, asymptotically normally distributed and asymptotically efficient estimators (Zellner and Theil, 1962) by estimating the covariance matrix from the residual covariance matrix of the 2SLS procedure. The third stage estimates the whole system of equations with generalized least squares.

To evaluate forest instant changes due to thinning, a control function is desirable. In this case, the quadratic mean diameter after thinning equation (Álvarez-González, 1997) was fitted in thinned intervals (Eq. (8)). The thinning function parameters were calculated by stepwise linear regression using PROC REG of SAS/STAT (SAS, 1990). In model application, the thinning must be defined by the percentage of removed basal area (Ge/Gbt). This data and the thinning equation (Eq. (8)) allow to estimate the stand characteristics after the thinning (quadratic mean diameter, basal area and density).

\[
dgat = c_0 + c_1 \times dgbt + c_2 \times \left( \frac{Ge}{Gbt} \right)^2 \times dgbt
\]

(8)

where $dgat$ is mean quadratic diameter after thinning, $dgbt$ is mean quadratic diameter before thinning, $Ge$ is the removed basal area, $Gbt$ is the stand basal area before thinning and $c_0$, $c_1$, and $c_2$, are parameters to be obtained.

In stands where thinning is not performed, a mortality rate (Eq. (9)) was calculated using non-linear regression, PROC NLIN of SAS/STAT (SAS, 1990).

\[
N_{i+5} = N_i \times \exp \left[ d_0 \times (t_{i+5} - t_i) \right]
\]

(9)

where $N_i$ is the stem stock at age $i$ and $d_0$ is the parameter to estimate.

Data from re-measured PSP were used to estimate the parameters of the growth and yield model, as well as mortality rate. Thinning control function was calculated using the second inventory when a low thinning was made in the PSP network.

2.2.2.2. Validation. Validation of the growth and yield model, as well as the dgat equation, was made by using the same statistics as in site index curves validation. The whole data set of thinning plots were used to validate the model. The mortality equation is
included in the model when thinning is not performed. To validate it only those intervals without thinning were used.

A simulation of the evolution of the bias and the RMSE in a long-term projection of basal area, volume and diameter after thinning is done for 5, 10, 15 and 25 years projection intervals.

3. Results

3.1. Site index curves

Results of the site index curves estimation are shown in Table 3. All the parameters are significant except for those of Bailey and Clutter function with free asymptote (Eq. (2.1)) which did not meet the convergence criterion, so it will not be taken into account for further discussion.

Errors measures and model efficiency values are similar in all models being slightly better for the Bailey and Clutter model with common asymptote and the McDill–Amateis function (over 97%). Models with common asymptote (Eqs. (1.2) and (2.2)) are unbiased. Some bias is shown in the Richards function with free asymptote, although the value is not very large (0.046 m).

With independent data (Table 4), Eqs. (1.1), (1.2) and (3) are quite precise, with relative AMRES lower than 5%, but statistically biased, resulting in overestimation of prediction when using Eq. (1.1) and underestimation with Eqs. (1.2) and (3). Eq. (2.2) is also quite precise and shows a little overestimation that it is not statistically biased. Moreover, the Bailey and Clutter model with common asymptote includes every feature a good model should possess, so we determine that this model is appropriate for our data and it is chosen as apt for evaluating site quality through site index estimation of P. pinaster Ait. in the Mediterranean distribution of the species. Fig. 1 shows the curves obtained when heights 9, 12, 15, 18, 21 and 24 m are reached at age 80. Stem analysis data and PSP data are also displayed.

3.2. Growth and yield model. Thinning control function and mortality rate

Results of the growth and yield model as well as the thinning control function and mortality rate are shown in Tables 5–7. Parameter \( a_i \) of basal area projection was not found to be significant in a first fit, so it was rejected and the system was fitted again showing low standards errors and great \( R^2 \) for the three equations of the system.

Validation results for growth and yield models, control function and mortality rate are shown in Table 8. For the compatible growth and yield system the volume projection shows the highest values of error statistics and lesser efficiency, although model efficiency of all equations are above 80% with independent data and errors are always below 2% so it is considered precise enough. However, the model

<table>
<thead>
<tr>
<th>Model</th>
<th>Free Parameter</th>
<th>Parameters estimated</th>
<th>SSE</th>
<th>MSE</th>
<th>EF</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>A</td>
<td>-0.0213 (0.00029)</td>
<td>1.3015 (0.0061)</td>
<td>2472.1 0.546 96.8 0.046**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td>k</td>
<td>26.929 (0.1264)</td>
<td>1.297 (0.0045)</td>
<td>2286.2 0.505 97.0 0.012**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.1</td>
<td>a</td>
<td>38.208** (828.7)</td>
<td>0.0062** (0.1396)</td>
<td>89565.9 19.81 -0.10 0.011**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.2</td>
<td>b</td>
<td>4.016 (0.0192)</td>
<td>-0.5031 (0.0051)</td>
<td>2047.9 0.452 97.3 0.009**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>1.360 (0.004)</td>
<td>2184.4 0.483 97.2 0.010**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In parenthesis approximate standard error. n.s.: no significance. Parameters as in Table 2. 

**P < 0.01.
shows some bias when independent data is used (Table 9). Quadratic mean diameter after thinning function is also quite precise with AMRES lower than 1 cm and high model efficiency, although it also shows some bias. Mortality rate is the only function of the model that is precise and unbiased.

Fig. 2 shows the long-term projection of volume, basal area and quadratic mean diameter. Volume is

Table 5
Fit results for growth and yield model

<table>
<thead>
<tr>
<th>Equation</th>
<th>SSE</th>
<th>MSE</th>
<th>RMSE</th>
<th>$R^2_{adjusted}$</th>
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<tbody>
<tr>
<td>5</td>
<td>2.2992</td>
<td>0.01000</td>
<td>0.1002</td>
<td>0.9731</td>
</tr>
<tr>
<td>6</td>
<td>1.1505</td>
<td>0.00498</td>
<td>0.0706</td>
<td>0.9470</td>
</tr>
<tr>
<td>7</td>
<td>1.7737</td>
<td>0.00746</td>
<td>0.0864</td>
<td>0.9701</td>
</tr>
</tbody>
</table>

Parameter | Estimate | Standard error | $t$ value | Probability |
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>$b_0$</td>
<td>1.916498</td>
<td>0.1066</td>
<td>17.97</td>
<td>0.0001</td>
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<tr>
<td>$b_1$</td>
<td>0.061317</td>
<td>0.00214</td>
<td>28.62</td>
<td>0.0001</td>
</tr>
<tr>
<td>$b_2$</td>
<td>-34.2235</td>
<td>1.3479</td>
<td>-25.39</td>
<td>0.0001</td>
</tr>
<tr>
<td>$b_3$</td>
<td>0.849985</td>
<td>0.0291</td>
<td>29.24</td>
<td>0.0001</td>
</tr>
<tr>
<td>$a_0$</td>
<td>4.746618</td>
<td>0.0406</td>
<td>116.88</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

SSE, sum of squares of the error; MSE, mean square error; RMSE, root mean square error.

Table 6
Analysis of variance for diameter after thinning (dgat) Eq. 8 ($R^2_{adjusted} = 0.9523$)

<table>
<thead>
<tr>
<th>Source</th>
<th>d.f.</th>
<th>SSE</th>
<th>MSE</th>
<th>$F$-value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>2</td>
<td>3987.49</td>
<td>1993.74</td>
<td>909.50</td>
<td>0.0001</td>
</tr>
<tr>
<td>Error</td>
<td>89</td>
<td>195.100</td>
<td>4.1921</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>91</td>
<td>4182.598</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Parameter | Estimate | Standard error | $T$ | Probability |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0$</td>
<td>1.56059</td>
<td>0.60907</td>
<td>2.56</td>
<td>0.0121</td>
</tr>
<tr>
<td>$c_1$</td>
<td>0.97928</td>
<td>0.02297</td>
<td>42.64</td>
<td>0.0001</td>
</tr>
<tr>
<td>$c_2$</td>
<td>0.66444</td>
<td>0.13771</td>
<td>4.82</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

SSE, sum of squares of the error; MSE, mean square error.
Table 7
Results of fitting for mortality rate Eq. 9

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Error standard</th>
<th>CI asymptotic at 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Asint.</td>
<td>Inferior</td>
<td>Superior</td>
</tr>
<tr>
<td>$d_0$</td>
<td>-0.00069</td>
<td>0.000132</td>
<td>-0.00095</td>
</tr>
</tbody>
</table>

CI asymptotic: asymptotic confidence interval for estimate parameter.

Table 8
Validation results for growth model, mortality and control function

<table>
<thead>
<tr>
<th>Equation</th>
<th>AMRES</th>
<th>Error (%)</th>
<th>RMSE</th>
<th>Error (%)</th>
<th>EF (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.060</td>
<td>1.23</td>
<td>0.011</td>
<td>0.23</td>
<td>95.19</td>
</tr>
<tr>
<td>6</td>
<td>0.030</td>
<td>0.82</td>
<td>0.005</td>
<td>0.14</td>
<td>98.43</td>
</tr>
<tr>
<td>7</td>
<td>0.062</td>
<td>1.12</td>
<td>0.012</td>
<td>0.22</td>
<td>81.29</td>
</tr>
<tr>
<td>8</td>
<td>0.966</td>
<td>4.61</td>
<td>0.176</td>
<td>0.84</td>
<td>87.42</td>
</tr>
<tr>
<td>9</td>
<td>9.59</td>
<td>0.91</td>
<td>15.21</td>
<td>1.44</td>
<td>99.33</td>
</tr>
</tbody>
</table>

AMRES: Absolute mean residuals; RMSE: root mean square errors; EF: model efficiency.

Table 9
Bias analysis of equations with fit and validation data

<table>
<thead>
<tr>
<th>Equation</th>
<th>Fit data value</th>
<th>Validation data value</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-0.00038 n.s.</td>
<td>0.04768**</td>
</tr>
<tr>
<td>6</td>
<td>-0.00027 n.s.</td>
<td>0.03995**</td>
</tr>
<tr>
<td>7</td>
<td>-0.00177 n.s.</td>
<td>0.01294**</td>
</tr>
<tr>
<td>8</td>
<td>-0.00007 n.s.</td>
<td>-0.92108**</td>
</tr>
<tr>
<td>9</td>
<td>-0.25 n.s.</td>
<td>-5.53 n.s.</td>
</tr>
</tbody>
</table>

n.s. no significance.
** P < 0.01.

underestimated in the two first intervals (5 and 10 years) whereas in long time projection, for 15 and 20 years, the volume is overestimated, resulting in increasing mean absolute values. Basal area projection is more precise than volume and the residuals are more close to zero. Finally, the quadratic diameter after thinning is overestimated during the whole projection.

![Residuals and absolute mean residuals (AMRES) for volume, basal area and quadratic mean diameter after thinning projection for short-term (5 years) and long-term (10, 15 and 20 years intervals).](image-url)
4. Discussion

The site index model has been developed using a difference equation approach. Among the five equations tested, the Bailey–Clutter expression with common asymptote (Eq. (2.2)) shows the best results with fitting and validation data. The Bailey–Clutter model is a modification of Korf’s equation and presents all the desirable characteristics. Among them, polymorphism is sometimes considered (Goelz and Burk, 1992), although the growth pattern of every species must be studied comparing different sort of equations. In this work, a model with common asymptote and polymorphic has been chosen as appropriate, in the same way as for Pinus pinea L., another Mediterranean pine species (Calama et al., 2003). However, the anamorphic Richards model with free asymptote is not far from the best result (the difference between absolute mean residual is only 0.04 m) and further studies by regions might be interesting in order to assess the anamorphic growth pattern of the species.

In Fig. 3, obtained site index curves are compared with the site index curves of a previous study (Pita, 1967) for Mediterranean subspecies and with the site index model proposed by Álvarez-González et al. (1999) for the Atlantic subspecies in the North-West of Spain. The different behaviour of the subspecies is clearly illustrated in the example. Both models for the Mediterranean subspecies present different forms especially for the best quality site at early ages.

Clutter’s model is the integral form of a basal area increment model and may be considered part of the growth model. The properties required by a transition function are attained by Clutter’s model: consistency, composition property, causality and compatibility (Sullivan and Clutter, 1972; García, 1994). In the model proposed for thinned loblolly pine stands (Pinus taeda L.) by Clutter (1963), the transition function depended on the state of the stand and the site index. The fact of non-significance of the parameter $a_1$ in the basal area growth equation for Mediterranean maritime pine coincides with Río and Montero, (2001), who reported the same result for Pinus sylvestris L. in Spain. This indicates that the state of the stand at a given moment is sufficiently characterised by initial state and that other variables such as site index may not be included in basal area projections for stands of these species. This is because basal area and age are good enough estimators of quality in stands where silviculture is applied next to forest dynamics with low thinning. Moreover, the range of ages does not include low values, where growth rate is higher and growth depends more on site index.

A model is often considered adequate when low values for statistics computed, no bias and high efficiency are found in the fitting procedure; however, a good model should perform properly also with independent data. In this sense, the set of functions developed in this work shows little bias, low error and high efficiency with independent data provided by managed plots. Values of residuals and absolute mean

![Fig. 3. Site index curves comparison. Double dashed line: subspecies atlantica Álvarez-González (1997) curves; dashed line: Pita (1967) curves; solid line curves: proposed in this paper.](image-url)
residuals in projection are not very high for volume or basal area, resulting an acceptable error (Table 8) in either, short and long-term projection. However the long-term projection of quadratic mean diameter is overestimated in near 4 cm and must be taken into account when planning thinning. The differences between observed and predicted values may be due to the different origin of the validations stands, which are plantations, and to the different silviculture applied.

Modelling is one of the main management keys due to its flexibility and utility in decision planning. The compatible growth and yield model developed provides a good indicator of the present and future main stand variables under different thinning and initial conditions, offering an essential tool to the managers. This approach seems adequate for the extensive management of *Pinus pinaster* stands in the Mediterranean area, in which input data are often measured at stand level. Nevertheless, if more detailed data were available, a tree-level approach could be interesting in order to study some aspects of sustainable management, such as structural diversity or fauna habitat assessment.

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References


